What is the meaning of the wave function ?

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Abstract

I will review the constraints that the no hidden variable theorems put on the interpretations of quantum mechanics and on the locality of the universe. I will also briefly explain how Bohm's theory gives a clear and objective meaning to the wave function and, at the same time, explains in the simplest possible way the most paradoxical aspects of quantum mechanics.

1. Introduction

Although it would be premature to advise a student to go into "foundations of quantum mechanics" if he or she does not enjoy independent means and needs to find a job, it should be said that this field has made remarkable (if slow) progress over the last decades. Several famous "'mainstream" physicists have devoted part of their activity to those problems. What is more important than this sociological aspect is that foundational statements no longer have a purely "philosophical" character: precise claims can be made and evaluated. concrete proposals can be ruled out experimentally, theorems can be proven. The goal of this paper is to review some of these results. starting with those that are "really proven" and moving from there to those that are merely reasonable <1>. I will sketch the main conceptual problems of quantum mechanics, but not, as is usually done, by stressing the problem of macroscopic superpositions (Schrödinger's cat paradox), but rather by dealing directly with the question in the title of this paper. In particular, I will show that a meaning that is often implicitly attributed to the wave function is simply untenable (Section 2). This is due to the so-called 'no hidden variable" theorems, which also have radical implications for locality (Section 3). Moreover, I will argue that the often-neglected theory of Bohm <2> offers a solution to the usual problems of (non-relativistic) quantum mechanics and reduces to some extent the puzzlement caused by the no hidden variable theorems (Section 4).

The wave function is the most fundamental concept of our most fundamental physical theory. Some explanation of what it means must be given, if only for pedagogical reasons. I shall suggest that there are at least two quite different meanings that can be given to the wave function.

Let me start by describing the usual quantum algorithm (leaving aside all subtleties related to continuous spectra): given an "observable" *A*, represented by a self-adjoint operator acting on the Hilbert space to which the wave function 4> belongs, we can write:

$$y = \Sigma\ ci\ f \qquad (1,1)$$

where the vectors fi are the eigenvectors of *A*, with eigenvalues λi :

$$A\ fi = li\ fI \qquad (1,2)$$

Then, y determines the coefficients Ci and the numbers [Ci]^2 are the probabilities that the eigenvalue li is found when the quantity corresponding to *A* is "measured".

But probabilities of what exactly? Giving different answers to that question leads to assigning different meanings to the wave function. I will discuss two possible answers, corresponding to what I will call the *literal* meaning and the *implicit* one. The failure to distinguish between the two is probably the source of much confusion. The literal meaning says that the probabilities refer solely to probabilities of experiments performed in the laboratory. The problem with this interpretation is that it does not allow us to say anything about the state of the world outside the laboratory. In particular, it is an unwarranted extrapolation to claim, for example, that $[y\ (x)]^2$ denotes the probability density of the position of the particle. It is rather, if one adheres to the literal interpretation, the probability density of *finding* the particle at *x* once a suitable measurement takes place. Before the measurement, it is neither here nor there. Note that I am not worried about the fact that the electron in an atom has a small probability of *being* behind the moon. I am worried about the fact that it has no probability of *being* anywhere.

Of course, the same argument applies to all other properties, like energy, momentum, angular momentum, spin etc...<3>. Particles have properties like their mass or their (total) spin, but these are generic properties, not individual ones. One may of course assign a wave function to an object outside the laboratory, but the point is that the wave function does not represent the state of the system, but only the probabilities of what would happen if that object was brought into a laboratory and interacted with an apparatus there. One cannot even say that the properties of the particle are "affected" by measurements. One should rather say that they are "created" by them or that the particle and the measuring device form an "inseparable whole". Actually, if one follows the literal interpretation, talking about particles or microscopic systems should be regarded as an abuse of language. The theory speaks only about the predictable and reliable behaviour of certain macroscopic objects called measuring devices <4>.

To see how serious the problem is, consider any scientific explanation. It always ultimately follows the reductionist line: properties of "objects" are explained in terms of those of their constituents, which are ultimately governed by quantum mechanics. But here, the theory becomes suddenly silent and talks only about the behaviour of macroscopic objects, and the reductionist line becomes a circle <5>.

At this point, the reader may think that I exaggerate and that this is not what the standard interpretation of quantum mechanics really says. Indeed, there is a second view, which I call the implicit one, namely that "measurement" really means measurement i.e. that experiments *reveal* rather than create preexisting properties of the system <6>. According to that view, particles *have properties* such as position, momentum, angular momentum and, when we measure an observable *A*, we simply discover the value that the system assigned to that observable independently of any measurement.

If one follows that interpretation, probabilities in quantum mechanics have a conceptual status rather similar to the one in classical physics: particles have properties such as spin, position or momentum, but the latter are unknown to us; hence, we use probabilities to describe that situation. The only novel feature <7> compared to classical probabilities is that these properties are unknowable, even in principle (or, at least, we can know only some of them at a time), because measuring one property changes the wave function (following the "collapse postulate") and hence changes the properties that are associated to operators that do not commute with the one being measured.

The implicit view is probably what lies behind familiar statements such as "the wave function does not represent the system but our knowledge of it" (or "quantum mechanics does not deal with Nature but with our knowledge of it"). According to that line of thought, the reduction of the wave function poses no particular problem. When we measure a system, we learn something about it, so our knowledge (i.e. the wave function) changes <8>. If that view was tenable, there would indeed be fewer reasons to worry. However, there are several difficulties with this view, the main one being that it is logically inconsistent as I shall explain in the next section.

2. The No Hidden Variable Theorems

There are several versions of such theorems. I will present one version here and a different one in the next section <9>. Let *A* be the set of self-adjoint operators on some Hilbert space (which may be taken of dimension four below).

Theorem 1 *There does not exist a map v:*

$$v : \boldsymbol{A} \rightarrow \mathbf{R} \tag{2.1}$$

such that

1) $\forall$ A $\in$ $\boldsymbol{A}$, v (A) $\in$ (set of eigenvalues of A) (2.2)

2) $\forall$ A, B $\in$ $\boldsymbol{A}$, with [A, B] = 0, v (A B) = v (A) v (B). (2.3)

Proof

We shall use operators given by the standard "x" and "y" Pauli matrices, for two "spins", $\boldsymbol{S}^{i}_{x}$, $\boldsymbol{S}^{i}_{y}$, I = 1, 2 where tensor products are tacitly understood $\boldsymbol{S}^{1}_{x} \equiv \boldsymbol{S}^{1}_{x} \otimes \mathbf{1}$, $\boldsymbol{S}^{2}_{x} \equiv \mathbf{1} \otimes \boldsymbol{S}^{2}_{x}$, etc. Those operators act on C4 The following identities are standard:

i) $(\boldsymbol{S}^{i}_{x})^2 = (\boldsymbol{S}^{i}_{y})^2 = \mathbf{1}$ (2,4)

for i = 1,2

ii) $\boldsymbol{S}^{i}_{x} \boldsymbol{S}^{i}_{y} = - \boldsymbol{S}^{i}_{y} \boldsymbol{S}^{i}_{x}$ (2,5)

for I = 1, 2

iii) $[\boldsymbol{S}^{1}_{a}, \boldsymbol{S}^{2}_{b}] = \mathbf{0}$ (2,6)

where a, b = x or y.

Now, consider the identity:

$$\boldsymbol{S}^{1}_{x} \boldsymbol{S}^{2}_{y} \boldsymbol{S}^{1}_{y} \boldsymbol{S}^{2}_{x} \quad \boldsymbol{S}^{1}_{x} \quad \boldsymbol{S}^{2}_{x} \boldsymbol{S}^{1}_{y} \boldsymbol{S}^{2}_{y} = -\mathbf{1} \qquad (2.7)$$

which follows, using first ii) and iii) above to move $\boldsymbol{S}^{1}_{x}$ in the product from the first place (starting from the left) to the fourth place, which involves one anticommutation and two commutations:

$$s^1_x s^2_y s^1_y s^2_x s^1_x s^2_x s^1_y s^2_y =$$

$$- s^2_y s^1_y s^2_x s^1_x s^1_x s^2_x s^1_y s^2_y$$

(2.8)

and then using repeatedly i) to see that the RHS of (8) equals **-1**.

Define the following set of operators:

A = $s^1_x s^2_y$

B = $s^1_y s^2_x$

C = $s^1_x s^2_x$

D = $s^1_y s^2_y$

X = *A.B*

Y = *C.D*

Then, observe

a [*A*, *B*]= 0

b [*C*, *D* = 0

g [*X*, *Y*]= 0

The identity (7) can be rewritten as

X.Y = **-1**. (2.9)

But, using assumption 2) of the Theorem *a) – γ)* and (6) above, we have:

a) *v(X.Y) = v(X)v(Y) = v(A)v(B)*

b) *v(A.B) = v(A)v(B)*

c) v (C.D) = v(C)v(D)

d) $v(A) = v(\boldsymbol{S}_x^1)\, v(\boldsymbol{S}_y^2)$

e) $v(B) = v(\boldsymbol{S}_y^1)\, v(\boldsymbol{S}_x^2)$

f) $v(C) = v(\boldsymbol{S}_x^1)\, v(\boldsymbol{S}_x^2)$

g) $v(D) = v(\boldsymbol{S}_y^1)\, v(\boldsymbol{S}_y^2)$

Since the only eigenvalue of the operator **-1** is -1, we have, combining (9) with assumption (1) in the Theorem and a)-g) above:

$$v(X.Y) = -1 = v(\boldsymbol{S}_x^1) v(\boldsymbol{S}_y^2) v(\boldsymbol{S}_y^1)\, v(\boldsymbol{S}_x^2) v(\boldsymbol{S}_x^1)\, v(\boldsymbol{S}_x^2)\, v(\boldsymbol{S}_y^1)\, v(\boldsymbol{S}_y^2) \tag{2.10}$$

where the RHS equals $v(\boldsymbol{S}_x^1)^2\, v(\boldsymbol{S}_y^2)^2\, v(\boldsymbol{S}_y^1)^2\, v(\boldsymbol{S}_x^2)^2$ since all factors in the product appear twice. But that last expression is manifestly positive (actually equal to +1), which is a contradiction.

Remarks

The symbol *v* is used for "value map"; indeed, one should think of *v(A)* as the putative value that the system assigns to the "observable" A *before measurement* (and that the latter merely *reveals).* The constraints 1) and 2) are justified on purely *empirical* grounds, entirely independent of the validity of the quantum theory as such. Indeed (for 1), the results of measurements are always eigenvalues; and (for 2), since *A* and B commute, one could (in principle) measure simultaneously *A, B* and *AB* and the results must satisfy (2.3).

The nonexistence of the map *v* means that measurements are, as one calls them, *contextual,* i.e. do not reveal preexisting properties of the system, but, in some sense, produce them; the word contextual refers to the fact that the result may depend not only on the microscopic system and the operator being "measured", but also on the "context", i.e. on some of the properties of the experimental set-up <10>. Hence, the implicit interpretation is untenable or at least has to be profoundly revised.

It should be stressed that we deal here with "experimental metaphysics" (as A. Shimony once called it), i.e. we are not merely concerned with the issue whether the value *v(A)* can be predicted (or controlled, or reproduced) by mere humans, but whether it is *logically consistent* simply to assume that it exists. It turns out that it is not, and that that result can be inferred from experimental constraints alone.

Finally, why is this called a "no hidden variable theorem"? As a result of some historical silliness", as Bell calls it ([2], p. 163, see also note 24, p.92), any variable that is not the wave function itself has been called a 'hidden variable". The name is somewhat silly because, if we are to go beyond the literal interpretation and, as explained above, it is necessary to do so, we have to assume that *something* exists besides the wave function. Bell has coined the word "beable" to refer to such objects (see [2], Chap.19). What the no hidden variable theorems say is simply that there cannot be beables corresponding to all observables (or even to certain classes of observables) <11>. Note also that the wave function itself is not 'visible"; like ah theoretical concepts, it is inferred.

3. Non locality and Bell's theorem

Here is a puzzle <12>: two persons, call them *X* and *Y*, leave a room through opposite doors; at that point, each is asked a question. The precise nature of the questions does not matter, but there are three possible questions (say, A, B and C). Each person must answer yes or no. This "experiment" is repeated many times, with sometimes the same question, sometimes different questions being asked at the two doors. The two persons are allowed to decide, before leaving the room, to follow any strategy they want, but not to communicate with each other, after they have heard the questions.

The statistics of answers have some strange properties. First of all, it turns out that when the two people are asked the same question they always give the same answer. Is that mysterious? Of course not; they simple decide, before leaving the room, to follow a certain strategy: for example, to both say 'yes' if the question is A, ‘no' if the question is B and ‘no' if the question is C. Altogether, there are $8 = 2^3$ different such strategies. Before proceeding further, the reader has to answer for himself or herself the following question: Is there any other way? Is there any way to account for the perfect correlations between the results at the two doors without assuming that the answers were predetermined (if we assume that the people cannot have *any communication whatsoever* with each other once the questions are asked)? I have never seen any suggestion of another possibility and I believe that if Bell's theorem is arguably the most widely misunderstood result in the history of physics, it is precisely because this question is not answered before proceeding further.

So, let us consider, for the time being, the assumption that the answers are predetermined and let us call $v_i(a) = \pm 1$, $i = X, Y$, $a = A, B, C$ those answers. These are "random variables", namely they may take different values when one repeats the "experiment". However, if one looks at the statistics of answers when *different questions* are asked at the two doors, one finds that the frequencies of the events in which the same answers are given is 1/4. And this, combined with the perfect correlations is strange. Indeed, a version of the no hidden variable theorems (similar to the one discussed in the previous section), known as Bell's theorem <13> , shows that this leads to a contradiction:

Theorem 2 *There does not exist random variables* $v_i(a)$, $i = X, Y$, $\alpha = A, B, C$ *such that*

1) $v_i(a) = \pm 1,$ (3,1)

2) $v_X(a) = v_Y(a), \quad \forall a,$ (3,2)

3) *Probability* $v_X(a) = v_Y(b) = 1/4 \qquad \forall a, b, \qquad a \neq b$ (3,3)

Proof

One has (with P for Probability):

$$P(v_X(A) = v_X(B)) + P(v_X(A) = (v_X(C)) + P(v_X(B) = (v_X(C)) \geq 1, \qquad (3.4)$$

because one of the three events in that equation must occur in each of the experiments, since $v_X(a)$ takes only two values. By 2), be get

$$P(v_X(A) = v_Y(B)) + P(v_X(A) = v_Y(C)) + P(v_Y(C)) \geq 1, \qquad (3,5)$$

and, using 3), we get 3/4 ³ 1, which is a contradiction.

Remarks

It is well known that there exist observations made with correlated photon pairs, which do reproduce those apparently "impossible" statistics; the "questions" correspond to three different angles along which the polarization is "measured" and the yes/no answers correspond to the two possible results (depending on the situation, we may have perfect correlations or perfect anticorrelations but that does not affect the crux of the argument). Finally, the 1/4 is simply the value of $\cos^2 60$ and comes from standard quantum mechanical calculations.

What is the conclusion of all this? We started from one crucial assumption: absence of "communication" between the two persons once they are asked the questions <14>. I will from now on revert to a less anthropomorphic language and call this assumption "locality" - assume that there is no causal connection whatsoever between the two wings of the experiment. Then, we are led to a contradiction, so that this assumption has to be dropped.

It is important to understand the logic of the argument: the perfect correlations plus the absence of communication (i.e. locality) between the two wings of the experiments, leads us to postulate the existence of the variables $v_i(a)$ (which are also "hidden" - see the discussion in section 2). However, merely assuming that those variables exist leads to a contradiction with the experimental results obtained when different questions are asked. To put it simply: locality plus perfect (anti)correlation implies hidden variables; however, the latter plus statistics when different angles are measured implies a contradiction. Both the perfect correlations and the statistics for different angles are empirical results; the theorem is a theorem, namely a logical deduction; the only assumption was the lack of "communication", or locality. Hence, locality has to be given up, period <15>.

It is not as is often assumed, a concern for realism, or for determinism or for hidden variables that is the source of the problem <16> The reason for this misunderstanding is probably historical: the first part of the argument (locality implies hidden variables) goes back essentially to Einstein, Podolsky and Rosen in 1935 [10]. But they did not put it this way and were certainly not interested in showing that the world is not local. On the contrary, they assumed (as if it was obvious) that the world is local and concluded that quantum mechanics is incomplete, namely that hidden variables must exist. As a logical reasoning, it was perfect.

But Bell's theorem was proven only in 1964, almost thirty years later. It showed that the hidden variables, whose existence was implied by the "obvious" assumption of locality made by EPR, led to a logical contradiction. However, between 1935 and 1964, the majority of the physics community became convinced that Bohr had satisfactorily answered Einstein [5]. I shall not discuss here what Bohr really had in mind, and how the majority of physicists actually read him. That is a fascinating problem for historians of science <17>.

The upshot is that Bell, who of course knew and understood the EPR argument, took it as his starting point and proceeded to disprove the existence of those hidden variables, hence of locality. But most of his readers did not understand the full logic of his argument and thought that Bell had merely refuted the existence of local hidden variables; since the EPR argument had simply been forgotten and since all concern for hidden variables were by then regarded as proof of a "lamentable addiction to metaphysics" <18>, Bell's result looked neither particularly spectacular nor particularly disturbing. That is, in a nutshell, why the majority of the physics community of the second half of this century underestimated the significance of one of the most startling results in the history of science.

4. Bohm's theory

In Bohm's theory, the wave function is not the complete description of the state of the system. The latter is given by both the usual wave function and the positions of all the particles:

$$(\psi, Q_1 \ldots, Q_N),$$

for a system of N particles. It is thus postulated that positions really exist, i.e. that these are the beables of the theory. The dynamics is also made of two pieces: one is the usual Schrödinger equation, which prescribes how the wave function evolves and the other is the guiding equation, which prescribes how the particles are guided by the wave:

$$\frac{d}{dt} Q_k = \frac{k}{m_k} I m \frac{y * \nabla_{qk} y}{y * y} (Q_1, \dots , Q_N) \qquad (4.1)$$

An important (though elementary) property of this system is *equivariance:* Assume that at some initial time t = O, we have a distribution of particles with density $r_O = |y_O|^2$. Then, at a later time the distribution of particles obtained by solving (4.1) will be $r_t = |y_t|^2$, where y_t is the solution of the Schrödinger equation with initial wave function y_O.

Using this fact, one shows that <19>, when one repeats a given experiment, the statistical results will agree with the quantum mechanical predictions if one assumes that the initial distribution for the microscopic system satisfies $r_O = |y_O|^2$. This is how the purely deterministic Bohm's theory explains the apparently "random" behaviour of quantum systems. One might of course worry about the nature and the justification of the statistical assumption at the initial time. This requires a long discussion, leading ultimately to assumptions on the initial state of the universe (see [8]), but the brief answer is that statistical mechanics also needs some initial statistical assumptions and that the latter also raise conceptual problems (that are, actually, much harder than the corresponding ones in Bohm's theory).

Let me now sketch how Bohm's theory clarifies the "paradoxes" arising from the no hidden variable theorems. First of all, there are no "hidden variables" in that theory other than positions. There is no value assigned by the system to various operators such as momentum, spin, angular momentum, etc. and that would be determined prior to those specific interactions with macroscopic devices misleadingly called "measurements". One can, in Bohm's theory, analyze how those interactions take place <20> and see that the result does not depend solely on the complete state (y, Q) of the microscopic system but also on the way the apparatus is set up. Hence, every "measurement" of anything but position is a *genuine interaction* between the system and the apparatus. It does not merely reveal preexisting properties of the system. In other words, Bohm's theory makes concrete and mathematically precise Bohr's intuition about the impossibility to separate the system and the apparatus.

Non locality is also easy to understand in Bohm's theory. The basic observation is that the wave function is a function defined on configuration space and not, as for example the electromagnetic field, on physical space. Consider then two particles and suppose that there is an external potential acting in the neighbourhood of, say, the origin and corresponding to the introduction of a measuring device acting on the first particle. The evolution of the wave-function will be affected by this potential through Schrödinger's equation; however, the wave function determines the trajectories of both particles through the guiding equation (4.1). Hence, the trajectory of the second particle will also be (indirectly) affected by the potential (that is by the measuring device) even if it happens to be very far from the origin. This gives some understanding of what goes on when polarization or spin "measurements" are performed on (anti)correlated pairs. The results, as Bell shows, are not determined before the interaction with the measuring device. And the perfect correlations are therefore due to a subtle form of "communication" between both sides of the experiment. The latter is made possible because the wave function connects distant parts of the universe through equation (4.1).

One final remark: the most common objections voiced against Bohm's theory are that it is "metaphysical" and "non local, hence incompatible with relativity". I will leave aside the first objection, which reflects some "positivistic" misunderstandings about the nature of physical theories. But the second objection is strange: after all, what Bell's theorem shows is that any theory that makes the correct experimental predictions must be non local. Being non local should be regarded, for a theory, as a virtue rather than a defect. And as far as compatibility with relativity is concerned, the problems faced by Bohm's theory are basically those that any theory will face as a consequence of Bell's theorem <21>.

5. Conclusions

I shall leave the last word to John Bell, one of the most lucid proponents of Bohm's theory. He explains that, when he was a student, he had read Born's book, *Natural Philosophy of Cause and Chance* [6J, where, on the basis of a misunderstanding of the significance of von Neumann's no hidden variable theorem, it was claimed that a deterministic theory underlying the quantum algorithm was impossible <22>. But, as he says, "in 1952, I saw the impossible done"; and that was Bohm's theory. He continues:

But then why had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing impossibility' proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm's version than to brand it as 'metaphysical' and 'ideological'? Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice? <23>

Notes :

1) A longer version of this paper, with complete proofs and detailed references to the historical background is in preparation.

2) Bohm [4] (1952>. See [2], [8] for a different presentation of that theory.

3) Except when the wave function happens to be an eigenstate of one of the observables, in which case one could argue that the probability of finding the corresponding eigenvalue being one, the system really "had" this value before it was measured.

4) 1f one follows this interpretation, quantum mechanics is entirely consistent with a view that is popular among some philosophers and sociologists of science: namely that science is true and objective, but only when its statements refer to what happens inside a laboratory. According to that view, scientists are not supposed to make claims referring to the outside world; in particular they should not talk about "laws of nature". Scientists should not dismiss such views as "nonsense", at least when the discussion is focused on quantum mechanics, unless they have a clear alternative to present. However, one should ask anyone who agrees with those views the obvious question: why build laboratories in the first place, if the experiments performed there do not allow us to know anything about what goes on in the world? One could answer that technology works, but then the question becomes: “why does it work?” And one is quickly led back to square one, namely understanding quantum mechanics.

5) Applied to evolution, this leads to particularly strange views: evolution works through mutations involving biochemical processes that are ultimately governed by quantum

mechanics. But any detailed description of these phenomena would have to refer to measurements made in present-day laboratories. As Wheeler puts it: "No phenomenon is a phenomenon until it is an observed phenomenon" [19]: See Bell ([2], chap. 14) for a critical discussion.

6) I call this view "implicit", because I believe that this is what many physicists think quantum mechanics means, although it is neither what the usual formulation implies nor, as we shall see, logically consistent.

7) Apart from the fact that the Schrödinger equation governs the evolution of the amplitude rather than that of the probability; this is true (and often emphasized) but is not relevant for the present discussion.

8) I believe that most physicists who do not worry about foundations of quantum mechanics often adopt this implicit view; however, a detailed historical analysis is needed in order to evaluate that conjecture.

9) See Mermin [17] for a nice discussion of those theorems and the links between them. However, Mermin's interpretation of their significance is, at least concerning locality, different from mine. The proofs are not included here, for lack of space, but are available from the preprint (at http://www.fyma.ucl.ac.be/reche/publications.html). I learned those theorems and their proofs, in the specific form given below, from Shelley Goldstein (private communications).

10) Actually, it would be better not to call them "measurements" at all, because the word does suggest that some objective property is being "observed". See Bell, [1], for a more detailed discussion.

11) This has rather serious consequences for the "decoherent histories" approach to foundations of quantum mechanics. See Goldstein [14] for a more detailed discussion.

12) This argument is due to Maudlin [16].

13) For a clear exposition of the link between those two types of theorems, see Mermin [17]. Both theorems are due to Bell, but the one of section 2 was discovered independently by Kochen and Specker (in a more complicated form). Penrose ([18), chapter 6, particularly endnote 14> gives also a nice discussion of this result.

14) If that assumption is dropped, there is no problem to account for the statistics: the two persons could agree to follow two different strategies: one if they are asked the same question and the other if the questions are different. And they simply tell each other which question is asked.

15) The natural question is whether this conflicts with relativity, since the experiments

indicate that the "communication" does not go at subluminal speeds. The short answer is that because of the "randomness" of the results on both sides, no superluminal transfer of information is made possible. But the issue is much more subtle than that. For a detailed discussion, see Maudlin [16].

16) See the discussion of the socks of Mr Bertlmann by Bell ([2], chap. 16), in particular note 10, p.157. For a remarkable misunderstanding of that very same example, see [11), p.172. See [13] for more examples of similar misunderstandings.

17) See Bell [2], p. 155, for a brief discussion of the Bohr-Einstein controversy. See also [15] for a good historical and conceptual discussion.

19) See [8] (and also [2], chap. 19) for a justification of this claim.

20) See [7], [12], [2], Chap.17.

21) See (2] (chap. 19), [3], [9] [16] for a more detailed discussion of relativity and of Bohmian quantum field theories.

22) See [13] for more examples of similar misunderstandings, going back to Von Neumann himself.

23) Bell. [2] p. 160.

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